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TWO PHASE MAGNETO HYDRODYNAMIC FLOW AND HEAT TRANSFER ON PROPAGATION OF WAVES.

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INTRODUCTION

In respect of fluid dynamics two phase flow generally refers to a flow in which two material phases are simultaneously present. In some cases, the two phases consists of different physical state of a single material such as bubbles of steam in water (Gas-Liquid). In other cases, the two phases each consists of a different material such as dust particle in air (Solid - gas).

Mathematical models of two phase flow are primarily applicable to situation mentioned as above. The two phases in some cases differ from each other with respect to their physical states. However, it is possible to use models to stimulate flows that involve mixture of two immiscible materials that exist in single physical state, for example, droplets of oil in water (liquid-liquid).

In the present paper we have studied the MHD flow and heat transfer in both the following two types of fluid mixture :-

(a) When the mixture is two fluids.
(b) When the mixture is of a gas and dust particles.

For the fluid in the two phase of first type, the fluid in the two-phase were assumed to be immiscible, incompressible. It is also assumed that the flow is steady, one-dimensional and fully developed. Further the two fluids are assumed to have different viscosities and thermal conductivities. We have investigated the upper phase and the lower phase of the two fluids was assumed to be electrically conducting and the conductivity of the two fluids is different. The transport properties of the two fluids were taken to be constant and the bounding plates are maintained at consists and equal temperature

The analytical solutions for velocities and temperature distributions are obtained and are evaluated numerically for different heights and viscosity ratios for the two fluids and for two values of the electric load parameter R_e. The results are plotted graphically. It is observed that in case of open circuit problem for negative values of the electric load parameter R_e, the effect of increasing the Hartmann number M is to accelerate the velocity and is to increase the temperature in contrast to the short case. Further it is also observed that with suitable values of the ratios of depths the velocity decreases but the temperature increases. Again if the ratio of the viscosities of the two fluids is increased the velocity increases but the temperature decreases and if the ratio of the thermal conductivities is increased the temperature can also be increased.

We have studied the magnetohydrodynamic heat transfer in two phase flow between two parallel plates in presence of a transverse magnetic field. We have assumed that both the phases are incompressible and to have constant transport properties and both the phases are taken to be electrically conducting. The plates were assumed to be maintained at constant and equal temperature. The investigation is expected to the useful in understanding the presence of slag layers on the heat transfer characteristic of coal-fired MHD generator. The boundaries of the generator channel will be assumed to be infinitely long, parallel plates. The exact solutions of velocity and temperature distributions are obtained. The results are shown graphically. Results are also presented for various depths, viscosities and thermalconductivities ratios for the two fluids and for two values of the electric field loading parameters.

For the second type, the effect of radiative heat transfer term on propagation of characteristic waves in two phase mixture of a gas and dust particle. When particle volume fraction appears as an additional variable is investigated and with the help of Laplace Transformation analytic solutions are obtained in characteristic plane.

2. FORMULATION OF THE PROBLEM AND EQUATIONS GOVERNING THE FLOW OF FIRST TYPE :-

The geometry under consideration shown in Figure -1 consists of two infinite parallel plates extending in the x and z directions. The region is $x \le y \le h_t$ is occupied by a fluid of viscosity μ_1 , electrical conductivity σ_1 and thermal conductivity k_1 and the region $-h_2 \le y \le 0$ is occupied by a layer of different (im-miscible) fluid of viscosity μ_2 , electrical conductivity σ_2 and thermal conductivity k_2. The transport properties of both fluids are assumed to be constant. The fluid flows in the direction of X in presence of a constant magnetic field of strength B_0 is applied in the direction of Y. The two bounding walls are maintained at constant temperature T_w. The fluid flow of both phases is assumed to be at a common pressure gradient $P = -\frac{\partial p}{\partial x}$, The fluid velocity and the magnetic field distributions are $\vec{V} = [u(y), 0, 0]$ and $\vec{B} = [0, B_0, 0]$ respectively. The flow is assumed to be steady, laminar, incompressible and fully developed.

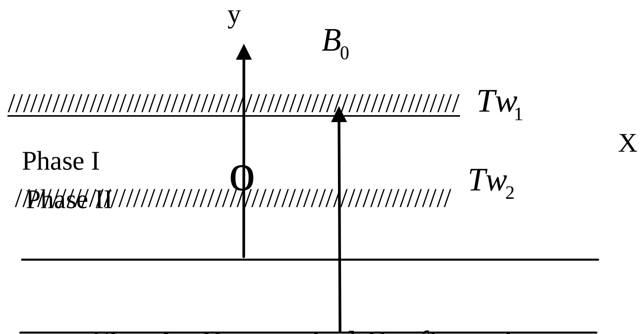

Fig : 1 : Geometrical Configuration

Under these assumptions as stated above, the governing equation of motion and equation of energy for two phases are-

$$\nabla . \vec{V} = 0 \qquad \text{-----------(2.1)}$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V} = -\left(\frac{1}{\rho}\right)\nabla p + \mu \nabla^2 \vec{V} + \frac{1}{\rho}\left(\vec{J} \text{ x } \vec{B}\right) + \vec{z} \qquad \text{-----------(2.2)}$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{V}.\nabla T\right) = k\nabla^2 T + \rho v \phi + \frac{J^2}{\sigma} \qquad \text{-----------(2.3)}$$

Here ϕ represent the dissipation function given by -

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 - \frac{3}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$

and the third term in the right hand side of equation (2.2) is the magnetic body force and $\vec{J}$ is the current density due to magnetic field defined by

$$\vec{J} = \sigma\left(\vec{E} + \vec{V} \text{ x } \vec{B}\right) \qquad \text{-----------(2.4)}$$

$\vec{Z}$ is the force due to buoyancy

$$Z = \beta g\left(T_0 - T\right). \qquad \text{-----------(2.5)}$$

The gravitational body force $\vec{Z}$ has been neglected in the equation (3.2).

Using the velocity and magnetic field distribution as stated above, the equation (2.1) to (2.3) are as follows :

The equation of motion and energy for the two phases reduces to

$$P + \mu \frac{\partial^2 u}{\partial y^2} - \sigma\left(E_z + uB_0\right)B_0 = 0 \qquad \text{-------------(2.6)}$$

$$\rho c_p u \frac{\partial T}{\partial x} = K \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + \sigma\left(E_z + uB_0\right)^2 \qquad \text{-------------(2.7)}$$

In the present problem, it is assumed that the two walls are maintained at constant and equation temperatures. Therefore the term involving $\frac{\partial T}{\partial x}$ in the energy equation (2.7) has been dropped out for such a condition. Thus the equation (2.7) becomes

$$k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + \sigma\left(E_z + uB_0\right)^2 = 0 \qquad \text{--------------(2.8)}$$

With these assumption, the governing equations of motion, energy and the corresponding boundary and interface conditions for the two phases are

Phase : 1

$$P + \mu_1 \frac{d^2 u_1}{dy_1^2} - \sigma_1\left(E_z + u_1 B_0\right)B_0 = 0 \qquad \text{---------------(2.9)}$$

$$k_1 \frac{d^2 T_1}{dy_1^2} + \mu_1 \left(\frac{du_1}{dy_1}\right)^2 + \sigma_1\left(E_z + u_1 B_0\right)^2 = 0 \qquad \text{--------------(2.10)}$$

Phase : 2

$$P + \mu_2 \frac{d^2 u_2}{dy_2^2} - \sigma_2\left(E_z + u_2 B_0\right)B_0 = 0 \qquad \text{--------------(2.11)}$$

$$k_2 \frac{d^2 T_2}{dy_2^2} + \mu_2 \left(\frac{du_2}{dy_2}\right)^2 + \sigma_2\left(E_z + u_2 B_0\right)^2 = 0 \qquad \text{--------------(2.12)}$$

The boundary and interface conditions on u_1 and u_2 are

$$u_1\left(+h_2\right) = 0,\; u_2\left(-h_2\right) = 0,\; u_1\left(0\right) = u_2\left(0\right),$$

$$\mu_1 \frac{du_1}{dy_1} = \mu_2 \frac{du_2}{dy_2} \text{ at } y = 0$$

If the walls are maintained at constant temperature the boundary conditions on T_1 and T_2 are given by

$$T_2\left(-h_2\right) = T_{w_2}, \qquad T_1\left(0\right) = T_2\left(0\right),$$

$$k_1 \frac{dT_1}{dy_1} = k_2 \frac{dT_2}{dy_2} \text{ at } y = 0 \qquad \text{--------------(2.14)}$$

Considering the non-dimensional terms :

$$u_1 = \left(\frac{u_1}{\bar{u}_1}\right), \quad y_1 = \frac{y_1}{h_1}, \quad u_2 = \left(\frac{u_2}{\bar{u}_2}\right), \quad y_2 = \frac{y_2}{h_2},$$

$$\mathrm{G} = \frac{P}{\left(\mu_1 \bar{u}_1 / h_1^2\right)},$$

$$\mathrm{R}_e = \frac{E_z}{\bar{u}_1 B_0}, \quad \theta_1 = \frac{T_1 - T_w}{\bar{u}_1^{\,2} \mu_1 / k_1}, \quad \theta_2 = \frac{T_2 - T_w}{\bar{u}_1^{\,2} \mu_1 / k_1} \qquad \text{-------------(2.15)}$$

Here T_w is the common wall temperature. Equations (2.9) equations (2.12) then transform to

$$\frac{d^2 u_1}{dy_1^2} + G - M_1^2 \left(R_e + u_1\right) = 0 \qquad \text{--------------(2.16)}$$

$$\frac{d^2 \theta_1}{dy_1^2} + \left(\frac{du_1}{dy_1}\right)^2 + M_1^2 \left(R_e + u_1\right) = 0 \qquad \text{--------------(2.17)}$$

$$\frac{d^2 u_2}{dy_2^2} + \frac{\alpha}{\beta^2} G + M_2^2 \left(R_e + u_2\right) = 0 \qquad \text{-------------(2.18)}$$

$$\frac{d^2 \theta_2}{dy_2^2} + \frac{\gamma}{\alpha}\left(\frac{du_2}{dy_2}\right)^2 + \frac{\gamma}{\alpha} M_2^2 \left(R_e + u_2\right)^2 = 0 \qquad \text{--------------(2.19)}$$

The subscripts 1 and 2 refers to the upper and lower phases respectively where M1 is the Hartman number in phases-I and M2 is the Hartman number in phase-II , which are the measures of the strengths of the applied magnetic field

In Phase-I, $M_1 = B_0 h_1 \sqrt{\frac{\sigma_1}{\mu_1}},$

In Phase-II $M_2 = B_0 h_2 \sqrt{\frac{\sigma_2}{\mu_2}},$

Re is the electric field aiding parameter,

α is a ratio of the viscosities of the two fluids $\alpha = \frac{\mu_1}{\mu_2},$

β is a ratio of the heights of the two fluids $\beta = \frac{h_1}{h_2}$

γ is a ratio of the thermal conductivities of the fluids, $\gamma = \frac{k_1}{k_2}$

δ is the ratio of the electrical conductivities of the two fluids $\delta = \frac{\sigma_1}{\sigma_2}$

G is the non dimensional pressure gradient.
The boundary conditions (2.13) and (2.14) reduces to

$$u_1(+1) = 0 \qquad \text{---------------(2.20)(a))}$$

$$u_2(-1) = 0 \qquad \text{---------------(2.20)(b))}$$

$$u_1(0) = u_2(0), \qquad \text{---------------(2.20)(c))}$$

$$\frac{du_1}{dy_1} = \left(\frac{\beta}{\alpha}\right)\frac{du_2}{dy_2} \quad \text{at } y = 0 \qquad \text{---------------(2.20)(d))}$$

$$\theta_1\left(+1\right) = 0, \qquad \text{---------------(2.21)(a))}$$

$$\theta_2(-1)=0, \quad \text{---------------(2.21)(b))}$$

$$\theta_1(0)=\theta_2(0), \quad \text{---------------(2.21)(c))}$$

$$\frac{d\theta_1}{dy_1}=\frac{\beta}{\gamma}\left(\frac{d\theta_2}{dy_2}\right) \text{ at } y=0 \quad \text{---------------(2.21)(d))}$$

3. SOLUTION OF EQUATIONS :-

Equations (2.16) and (2.18) can be solved simultaneously for velocities u_1 and u_2 subject to the conditions (2.20a) to (2.20d). The exact solutions for the velocities of the- phases are given by :

$$u_1[y_1]=c_1\cosh[M_1y_1]+\frac{c_3\sinh[M_1y_1]}{M_1}-\frac{G_1}{M_1^2}, \quad \text{---------------(3.1)}$$

$$u_2[y_2]=c_2\cosh[M_2y_2]+\frac{c_4\sinh[M_2y_2]}{M_2}-\frac{G_2}{M_2^2}, \quad \text{---------------(3.2)}$$

where $G_1=M_1^2R_e-G;\ G_2=M_2{}^2R_e-G\dfrac{\alpha}{\beta^2};$

$$k_1=M_1M_2\left(\alpha\cos[M_1]\sinh[M_2]M_1+\beta\cosh[M_2]\sinh[M_1]M_2\right);$$

$$c_1=\frac{1}{k_1M_1}\left(-2\beta\sinh[M_1]\sinh\left[\frac{[M_2]}{2}\right]^2G_2M_1{}^2 + G_1M_2\left(\alpha\sinh[M_2]M_1+\beta\cosh[M_2]\sinh[M_1]M_2\right)\right)$$

$$c_2=\frac{1}{K_1M_2}\left(-\alpha\cos[M_1]-1\right)\sinh[M_2]G_1M_2{}^2 + G_1M_2{}^2\left(\alpha cos\,h[M_1]\sinh[M_2]M_1+\beta\sinh[M_1]M_2\right)$$

$$c_3=\frac{\beta\left(2\cosh[M_1]\sinh\left[\dfrac{M_2}{2}\right]^2G_2M_1{}^2-\left(\cosh[M_1]-1\right)\cosh[M_2]G_1M_2{}^2\right)}{k_1}$$

$$c_4=\frac{\beta\left(2\cosh[M_2]\sinh\left[\dfrac{M_2}{2}\right]^2G_2M_1{}^2-\left(\cosh[M_1]-1\right)\cosh[M_1]G_1M_1{}^2\right)}{k_1}$$

Once the velocity distributions are known, the temperature distributions for the regions can be determined by using the equations (2.10), (2.12) and (2.21)-(2.23). The temperature distribution for Phase-I is given by

$$\theta_1[y_1]=\frac{1}{4M_1{}^2}\left(\begin{array}{l}\cosh[2M_1y_1]c_3{}^2+4\sinh[M_1y_1]c_3\left(\cosh[M_1y_1]c_1 2k_5\right)M_1+\\ M_1{}^2(-4c_5-4c_7y_1)+\cosh[2M_1\,y_1c_1{}^2+8\cosh[M_1y_1]c_1k_5+2k_5{}^2M_1{}^2y_1{}^2\end{array}\right)$$

---------------(3.3)

and for phase II it will be given by

$$\theta_2[y_2] = -\frac{1}{4\alpha M_2^2}\left(\gamma \cosh[2M_2\, y_2]c_4^2 + 4\gamma \sin[2M_2\, y_2]c_4\left(\cos\gamma[M_2\, y_2]c_2\right.\right.$$
$$\left.+2K_6\right)M_2 + M_2^2\left(-4\alpha c_6 - 4\alpha c_8\, y_2 + \gamma \cosh[2M_2\, y_2]c_2^2\right.$$
$$\left.\left.+8\gamma \cosh[M_2\, y_2]c_2 k_6 + 2\gamma k_6^2 M_2^2\, y_2^2\right)\right) \qquad \text{---------------(3.4)}$$

Where $k_5 = R_e - \frac{G_1}{M_1^2}; \quad k_6 = R_e - \frac{G_2}{M_2^2}; \quad k_7 = \frac{1}{4\alpha(\beta+\gamma)M_1^2 M_2^2};$

$$c_5 = k_7\left(2\beta\gamma \sinh[M_2]^2 c_4^2 M_1^2 + 2\beta\gamma c_4 M_1^2 M_2\left(-c_2\left(\sinh[2M_2]-2M_2\right)+4k_6\left(M_2-\sinh[M_2]\right)\right)\right.$$
$$+M_2^2(\alpha(\beta+\gamma\cosh[2M_1]c_3^2 + 2\alpha\gamma c_3 M_1(\sinh[2M_1]c_1 + 4\sinh[M_1]k_5 - 2c_1M_1 - 4k_5M_1)$$
$$+M_1^2(\alpha\beta c_1^2 + \alpha\gamma\cosh[2M_1]c_1^2 - \beta\gamma c_2^2(1-\cosh[2M_2]) + 8\alpha c_1 k_5(\beta+\gamma\cosh[M_1])$$
$$-8\beta\gamma c_2 k_6 + 8\beta\gamma\cosh[M_2]c_2k_6 + 2\alpha\gamma k_5^2 M_1^2 + 2\beta\gamma k_6^2 M_2^2)))$$
$$c_6 = k_7(\gamma((\gamma+\beta\cosh[2M_2])c_4^2M_1^2 + 2\beta c_4 M_1^2 M_2(-c_2(\sinh[2M_2]-2M_2)+4k_6(M_2-\sinh[M_2]))$$
$$+M_2^2(2\alpha\sinh[M_1]^2 c_3^2 + 2\alpha c_3 M_1(\sinh[2M_1]c_1 + 4\sinh[M_1]k_5 - 2c_1M_1 - 4k_5M_1)$$
$$+M_1^2(-\alpha c_1^1 + \alpha\cosh[2M_1]c_1^2 + \gamma c_2^2 + \beta\cosh[2M_2]c_2^2 - 8\alpha c_1k_5 + 8\alpha\cosh[M_1]c_1k_5$$
$$+8\gamma c_2k_6 + 8\beta\cosh[M_2]c_2k_6 + 2\alpha k_5^2M_1^2 + 2\beta k_6^2M_2^2))))$$
$$c_7 = k_7(-2\beta\gamma\sinh[M_2]^2 c_4^2M_1^2 + 2\beta\gamma c_4M_1^2(c_2(\sinh[2M_2]-2M_2)+4k_6(\sinh[M_2]-M_2)))M_2$$
$$+M_2^2(2\alpha\beta\sinh[M_1]^2 c_3^2 + 2\alpha c_3M_1(\beta\sinh[2M_1]c_1 + 4\beta\sinh[M_1]k_5 + 2\gamma c_1M_1 - 4k_5M_1)$$
$$+\beta M_1^2(-\alpha c_1^2 + \alpha\cosh[2M_1]c_1^2 + \gamma c_2^2 - \gamma\cosh[2M_2]c_2^2 - 8\alpha c_1k_5 + 8\alpha\cosh[M_1]c_1k_5$$
$$+8\gamma c_2k_6 - 8\gamma\cosh[M_2]c_2k_6 + 2\alpha k_5^2M_1^2 - 2\gamma k_6^2M_2^2)))$$
$$c_8 = k_7(\gamma(-2\gamma\sinh[M_2]^2 c_4^2M_1^2 + 2c_4M_1^2\, M_2(4k_6(\gamma\sinh[M_2]+\beta M_2)+c_2(\gamma\sinh[M_2]+2\beta M_2))$$
$$+M_2^2(2\alpha\sinh[M_1]^2 c_3^2 + 2\alpha c_3M_1(\sinh[2M_1]c_1 + 4\sinh[M_1]k_5 - 2c_1M_1 - 4k_5M_1)$$
$$+M_1^2(-\alpha c_1^1 + \alpha\cosh[2M_1]c_1^2 + \gamma c_2^2 - \gamma\cosh[2M_2]c_2^2 - 8\alpha c_1k_5 + 8\alpha\cosh[M_1]c_1k_5$$
$$+8\gamma c_2k_6 - 8\gamma\cosh[M_2]c_2k_6 + 2\alpha k_5^2M_1^2 - 2\gamma k_6^2M_2^2))))$$

Equation (3.1) to equation (3.4) are evaluated numerically for different values of parameters and the results so obtained are presented graphically.

4. <u>RESULT AND DISSCUSSION</u> :-

We have observed that the electromagnetic force is to accelerate the flow. If the ratio of viscosities of two fluids increases then the velocities also increases slowly but in case of temperature distribution if the ratio viscosities of two fluids decreases then the temperature increases very rapidly. It is observed that if β decreases the velocity increases first very slowly then rapidly, but in case of temperature distribution velocity gradually increases with the increasing value of β.

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